**MATHEMATICS**

**PAGEMAKER10**

**Circle**

Q1. Locus of the point given by the equations x = $\frac{2at}{1+t^{2}}$ , y = $\frac{a(1-t^{2})}{1+t^{2}}$

(–1 $\leq $ t $\leq $ 1) is a

(a) Straight line

(b) Circle

(c) Ellipse

(d) Hyperbola

L1Difficulty1

Qtag Mathematics

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Q2. The equation of the circle with origin as centre passing through the vertices of an equilateral triangle whose median m of length 3a is

(a) x2 + y2 = 9a2

(b) x2 + y2 = 16a2

(c) x2 + y2 = a2

(d) None of these

L1Difficulty1

Qtag Mathematics

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Q3. If the line 3x + 4y – 1 = 0, touches the circle (x – 1)2 + (y – 2)2 = r2, then the value of r will be

(a) 2

(b) 5

(c) $\frac{12}{5}$

(d) $\frac{2}{5}$

L1Difficulty1

Qtag Mathematics

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Q4. The locus of a point which divides the join of A(–1, 1) and a variable point P on the circle x2 + y2 = 4 in the ratio 3:2 is :

(a) 25(x2 + y2) + 20(x – y) + 28 = 0

(b) 25(x2 + y2) + 20(x – y) – 28 = 0

(c) 20(x2 + y2) + 25(x – y) + 28 = 0

(d) None of these

L1Difficulty1

Qtag Mathematics

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Q5. The abscissa of A and B are the roots of the equation x2 + 2ax – b2 = 0, and their ordinates are the roots of the equation y2 + 2py – q2 = 0. The equation of the circle with AB as diameter.

(a) x2 + y2 + 2ax + 2py – b2 – q2 = 0

(b) x2 + y2 + 2ax + py – b2 – q2 = 0

(c) x2 + y2+ 2ax + 2py + b2 + q2 = 0

(d) None of these

L1Difficulty1

Qtag Mathematics

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Q6. Chord of contact of the point (3, 2) w.r.t. the circle x2 + y2 = 25 meets the coordinate axes in A and B. The circumcentre of triangle OAB is

(a) $\left(\frac{25}{4}, \frac{25}{6}\right)$

(b) $\left(\frac{2}{50}, \frac{3}{50}\right)$

(c) $\left(\frac{25}{6}, \frac{25}{4}\right)$

(d) None of these

L1Difficulty1

Qtag Mathematics

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Q7. The normal at the point (3, 4) on a circle cuts at the point (–1, –2). Then the equation of the circle is

(a) x2 + y2 + 2x – 2y – 13 = 0

(b) x2 + y2 – 2x – 2y – 11 = 0

(c) x2 + y2 – 2x + 2y + 12 = 0

(d) x2 + y2 – 2x – 2y + 14 = 0

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q8. The tangents are drawn from the points (4, 5) to the circle x2 + y2 – 4x – 2y – 11 = 0. The area of quadrilateral formed by these tangents and radii, is

(a) 15 sq. units

(b) 75 sq. units

(c) 8 sq. units

(d) 4 sq. units

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q9. If a straight line through C(–$\sqrt{8}, \sqrt{8}$) making an angle of 135° with the x – axis cuts the circle x = 5 Cos $θ$, y = 5 Sin $θ$ at points A and B, then the length of AB is

(a) 3

(b) 7

(c) 10

(d) None of these

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

Q10. The number of common tangents to the circles x2 + y2 = 4 and x2 + y2 = 4 and x2 + y2 – 6x – 8y = 24 is

(a) 0

(b) 1

(c) 3

(d) 4

L1Difficulty1

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (b)

Sol.

Suppose x = $\frac{2at}{1+t^{2}}$ and y = $\frac{a(1-t^{2})}{1+t^{2}}$

Squaring ad adding both,

we get x2 + y2 = a2

S2. Ans. (d)

Sol.

Centre (0, 0), radius = 3a × $\frac{2}{3}$ = 2a

Hence circle x2 + y2 = 4a2 as centroid divides median is the ratio of 2:1.

S3. Ans. (a)

Sol.

If the line 3x + 4y – 1 = 0 touches the circle (x – 1)2 + (y – 2)2 = r2, then the perpendicular from centre of circle on line is equal to the radius of circle i.e. $\left[\frac{3+8-1}{5}\right]$ = r or r = 2

S4. Ans. (b)

Sol.

Suppose a point on circle is B(x1, y1) and that which divides A and B, in 3:2 is P given by

h = $\frac{-2+3x\_{1}}{5}$, k = $\frac{2+3y\_{1}}{5}$ or $\frac{5h+2}{3}$ = x1

$\frac{5k-2}{2}$ = y1

As (x1 y1) lies on circle x2 + y2 = 4, we get on substituting, 25(x2 + y2) + 20(x – y) – 28 = 0

S5. Ans. (a)

Sol.

Let A (x1, y1) and B (x2, y2), then

x1 + x2 = –2a

x1x2 = –b2

y1 + y2 = 2p

y1 y2 = –q2

Now find centre and radius and hence the equation of circle.

S6. Ans. (d)

Sol.

Since S(3, 2) = 9 + 4 – 25 < 0, therefore (3, 2) lies inside the circle. So these exists no chord of contact and hence $Δ$OAB does not exist.

S7. Ans. (b)

Sol.

Since normal passes through the centre of the circle.

$∴$ the required circle is the circle with ends of diameter as (3, 4) and (–1, –2)

$∴$ Its equations is (x – 3) (x + 1) + (y – 4) (y + 2) = 0

$⇒$ x2 + y2 – 2x – 2y – 11 = 0

S8. Ans. (c)

Sol.

Length of each tangent

L2 = (4)2 + (5)2 – (4 × 4) – (2 × 5) – 11

L = 2

r = $\sqrt{2^{2}+1^{2}-(-11)}$

r = 4

Area = L × r = 8 sq. units

S9. Ans. (c)

Sol.

Line AB is x + y = 0, which is diameter of the circle x2 + y2 = 25. Its length = 2r = 10

S10. Ans. (b)

Sol.

Circles S1 = x2 + y2 = 22, S2 = (x – 3)2 + (y – 4)2 – 72

$∴$ Centre C1 = (0, 0), C2 = (3, 4)

and radii r1 = 2; r2 = 7, $∴$ C1 C2 = 5, r2 – r1 = 5

i.e. Circles touch internally, Hence there is only one common tangent.

**LEVEL-II**

Q1. The equation of the circle which touches both the axes and whose radius is $a,$ is

(a) $x^{2}+y^{2}-2ax-2ay+a^{2}=0$

(b) $x^{2}+y^{2}+ax+ay-a^{2}=0$

(c) $x^{2}+y^{2}+2ax+2ay-a^{2}=0$

(d) $x^{2}+y^{2}-ax-ay+a^{2}=0$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q2. The area of the circle whose centre is at (1, 2) and which passes through the point (4, 6) is

(a) $5π$

(b) $10π$

(c) $25π$

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q3. The centres of the circles $x^{2}+y^{2}=1, x^{2}+y^{2}+6x-2y=1,$ $x^{2}+y^{2}-12x+4y=1$ are

(a) Same

(b) Collinear

(c) Non-collinear

(d) None of these

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q4. The equation of a circle which touches both axes and the line $3x-4y+8=0$ and whose centre lies in the third quadrant is

(a) $x^{2}+y^{2}-4x+4y-4=0$

(b) $x^{2}+y^{2}-4x+4y+4=0$

(c) $x^{2}+y^{2}+4x+4y+4=0$

(d) $x^{2}+y^{2}-4x-4y-4=0$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q5. If one end of a diameter of the circle $x^{2}+y^{2}-4x-6y+11=0$ be (3, 4), then the other end is

(a) $(0, 0)$

(b) (1, 1)

(c) $(1, 2)$

(d) $(2, 1)$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q6. If the equation $px^{2}+\left(2-q\right)xy+3y^{2}-6qx+30y+6q=0$ represents a circle, then the values of $p $and $q$ are

(a) 3, 1

(b) 2, 2

(c) 3, 2

(d) 3, 4

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q7. The equation of the circle passing through the origin and cutting intercepts of length 3 and 4 units from the positive axes, is

(a) $x^{2}+y^{2}+6x+8y+1=0$

(b) $x^{2}+y^{2}-6x-8y=0$

(c) $x^{2}+y^{2}+3x+4y=0$

(d) $x^{2}+y^{2}-3x-4y=0$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q8. If the length of tangent drawn from the point (5, 3) to the circle $x^{2}+y^{2}+2x+ky+17=0$ be 7, then $k=$

(a) 4

(b) $-4$

(c) $-6$

(d) 13/2

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q9. The line $lx+my+n=0$ will be a tangent to the circle $x^{2}+y^{2}=a^{2}$ if

(a) $n^{2}\left(l^{2}+m^{2}\right)=a^{2}$

(b) $a^{2}\left(l^{2}+m^{2}\right)=n^{2}$

(c) $n\left(l+m\right)=a$

(d) $a\left(l+m\right)=n$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

Q10. The angle between the two tangents from the origin to the circle $x-7)^{2}+\left(y+1\right)^{2}=25$ is

(a) $0$

(b) $\frac{π}{3}$

(c) $\frac{π}{6}$

(d) $\frac{π}{2}$

L3Difficulty3

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (a)

Sol.

Required equation is $\left(x-a\right)^{2}+\left(y-a\right)^{2}=a^{2}$

$⇒$ $x^{2}+y^{2}-2x-2ay+a^{2}=0$

S2. Ans. (c)

Sol.

Obviously radius = $\sqrt{\left(1-4\right)^{2}+\left(2-6\right)^{2}}=5$

Hence the area is given by $πr^{2}=25π sq.units.$

S3. Ans. (b)

Sol.

Centres are $\left(0, 0\right), (-3, 1)$, and $(6, -2)$ and a line passing through any two points say $(0, 0)$ and $(-3, 1)$ is $y=-\frac{1}{3}x$ and point ($6, -2)$ lies on it. Hence points are collinear.

S4. Ans. (c)

Sol.

The equation of circle in third quadrant touching the coordinate axes with centre $(-a, -a)$ and radius $'a'$ is $x^{2}+y^{2}+2ax+2ay+a^{2}=0$ and we know

$\left|\frac{3\left(-a\right)-4\left(-a\right)+8}{\sqrt{9+16}}\right|=a⇒a=2$

Hence the required equation is

$x^{2}+y^{2}+4x+4y+4=0.$

**Trick :** Obviously the centre of the circle lies in III quadrant, which is given by $\left(c\right).$

S5. Ans. (c)

Sol.

Centre is (2, 3). One end is (3, 4).

$P\_{2}$ divides the join of $P\_{1}$ and $O$ in ratio of $2 :1.$

Hence $P\_{2}$ is $\left(\frac{4-3}{2-1},\frac{6-4}{2-1}\right)=\left(1, 2\right).$

S6. Ans. (c)

Sol.

In the equation of circle, there is no term containing $xy$ and coefficient of $x^{2}$ and $y^{2}$ are equal. Therefore $2-q=0⇒q=2$ and $p=3.$

S7. Ans. (d)

Sol.

Obviously the centre of the circle is $\left(\frac{3}{2}, 2\right).$

Therefore, the equation of circle is

$\left(x-\frac{3}{2}\right)^{2}+\left(y-2\right)^{2}=\left(\frac{5}{2}\right)^{2}⇒x^{2}+y^{2}-3x-4y=0.$

S8. Ans. (b)

Sol.

According to the condition,

$\sqrt{\left(5\right)^{2}+\left(3\right)^{2}+2\left(5\right)+k\left(3\right)+17}=7$

$⇒61+3k=49⇒k=-4.$

S9. Ans. (b)

Sol.

Line $y=mx+c$ is tangent, if $c=\pm a\sqrt{1+m^{2}}$.

Now $lx+my+n=0$ or $y=-\frac{1}{m}x-\frac{n}{m}$ is tangent, if

$-\frac{n}{m}+\pm a\sqrt{1+\left(\frac{1}{m}\right)^{2}}$ or $n^{2}=a^{2}\left(m^{2}+l^{2}\right).$

S10. Ans. (d)

Sol.

Any line through (0, 0) be $y-mx=0$ and it is a tangent to circle $\left(x-7\right)^{2}+\left(y+1\right)^{2}=25$, if

$\frac{-1-7m}{\sqrt{1+m^{2}}}=5⇒m=\frac{3}{4},-\frac{4}{3} .$

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Therefore, the product of both the slopes is $-1.$

$i.e.,\frac{3}{4}×-\frac{4}{3}=-1.$

Hence the angle between the two tangents is $\frac{π}{2}$ .

**LEVEL-III**

Q1. A pair of tangents are drawn from the origin to the circle $x^{2}+y^{2}+20\left(x+y\right)+20=0.$ The equation of the pair of tangent is

(a) $x^{2}+y^{2}+10xy=0$

(b) $x^{2}+y^{2}+5xy=0$

(c) $2x^{2}+2y^{2}+5xy=0$

(d) $2x^{2}+2y^{2}-5xy=0$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q2. If $OA$ and $OB$ be the tangents to the circle $x^{2}+y^{2}-6x-8y+21=0$ drawn from the origin $O$, then $AB=$

(a) 11

(b) $\frac{4}{5}\sqrt{21}$

(c) $\sqrt{\frac{17}{3}}$

(d) None of these

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q3. Equation of the pair of tangents drawn from the origin to the circle $x^{2}+y^{2}+2gx+2fy+c=0$ is

(a) $gx+fy+c(x^{2}+y^{2})$

(b) $\left(gx+fy\right)^{2}=x^{2}+y^{2}$

(c) $\left(gx+fy\right)^{2}=c^{2}(x^{2}+y^{2})$

(d) $\left(gx+fy\right)^{2}=c(x^{2}+y^{2})$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q4. If the line $y=mx+c$ be a tangent to the circle $x^{2}+y^{2}=a^{2}$, then the point of contact is

(a) $\left(\frac{–a^{2}}{c},a^{2}\right)$

(b) $\left(\frac{a^{2}}{c},\frac{-a^{2}m}{c}\right)$

(c) $\left(\frac{–a^{2}m}{c},\frac{a^{2}}{c}\right)$

(d) $\left(\frac{–a^{2}c}{m},\frac{a^{2}}{m}\right)$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q5. If the length of the tangent from any point on the circle $\left(x-3\right)^{2}+\left(y+2\right)^{2}=5r^{2}$ to the circle $\left(x-3\right)^{2}+\left(y+2\right)^{2}=r^{2}$ is 16 units, then the area between the two circles in sq. units is

(a) 32$π$

(b) 256$π$

(c) 8$π$

(d) 16$π$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q6. A point inside the circle $x^{2}+y^{2}+3x-3y+2=0$ is

(a) $(-1, 3)$

(b) $(-2, 1)$

(c) $(2, 1)$

(d) $(-3, 2)$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q7. $y=mx$ is a chord of a circle of radius $a$ and the diameter of the circle lies along x-axis and one end of this chord in origin. The equation of the circle described on this chord as diameter is

(a) $\left(1+m^{2}\right)\left(x^{2}+y^{2}\right)-2ax=0$

(b) $\left(1+m^{2}\right)\left(x^{2}+y^{2}\right)-2a\left(x+my\right)=0$

(c) $\left(1+m^{2}\right)\left(x^{2}+y^{2}\right)+2a\left(x+my\right)=0$

(d) $\left(1+m^{2}\right)\left(x^{2}+y^{2}\right)-2a\left(x-my\right)=0$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q8. The locus of the middle points of those chords of the circle $x^{2}+y^{2}=4$ which subtend a right angle at the origin is

(a) $x^{2}+y^{2}-2x-2y=0$

(b) $x^{2}+y^{2}=4$

(c) $x^{2}+y^{2}=2$

(d) $\left(x-1\right)^{2}+\left(y-2\right)^{2}=5$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q9. The equation of the chord of the circle $x^{2}+y^{2}=a^{2}$ having $(x\_{1}, y\_{1})$ as its mid-point is

(a) $xy\_{1}+yx\_{1}=a^{2}$

(b) $x\_{1}+y\_{1}=a$

(c) $xx\_{1}+yy\_{1}=x\_{1}^{2}+y\_{1}^{2}$

(d) $xx\_{1}+yy\_{1}=a^{2}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

Q10. The length of the common chord of the circles $x^{2}+y^{2}+2x+3y+1=0$ and $x^{2}+y^{2}+2x+3y+1=0$ and $x^{2}+y^{2}+4x+3y+2=0$, is

(a) $\frac{9}{2}$

(b) $2\sqrt{2}$

(c) $3\sqrt{2}$

(d) $\frac{3}{2}$

L5Difficulty5

Qtag Mathematics

Qcreator Pagemaker10

**Solutions**

S1. Ans. (c)

Sol.

Equation of pair of tangents is given by $SS\_{1}=T^{2}.$ Here $S=x^{2}+y^{2}+20\left(x+y\right)+20, S\_{1}=2$0

$T=10\left(x+y\right)+20$

$∴SS\_{1}=T^{2}$

$⇒20\left\{x^{2}+y^{2}+20\left(x+y\right)+20\right\}=10^{2}\left(x+y+2\right)^{2}$

$⇒4x^{2}+4y^{2}+10xy=0⇒2x^{2}+2y^{2}+5xy=0.$

S2. Ans. (b)

Sol.

Here the equation of $AB$ (chord of contract) is

$x x\_{1}+y y\_{1}+g\left(x+x\_{1}\right)+f\left(fy+y\_{1}\right)+21=0$

$∴0+0-3\left(x+0\right)-4\left(y+0\right)+21=0$

$$⇒3x+4y-21=0$$

 …(i)

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$CM=$ perpendicular distance from (3, 4) to line (i) is

$\frac{3×3+4×4-21}{\sqrt{9+16}}=\frac{4}{5}$

$AM=\sqrt{AC^{2}-CM^{2}}=\sqrt{4-\frac{16}{25}}=\frac{2}{5}\sqrt{21}$

$∴AB=2AM=\frac{4}{5}\sqrt{21}$ .

S3. Ans. (d)

Sol.

Equation of pair of tangents is $SS\_{1}=T^{2}$, where $T=xx\_{1}+yy\_{1}+g\left(x+x\_{1}\right)+f\left(y+y\_{1}\right)+c $

$⇒c\left(x^{2}+y^{2}+2gx+2fy+c\right)=\left(gx+fy+c\right)^{2}$

$⇒c\left(x^{2}+y^{2}\right)=\left(gx+fy\right)^{2}.$

S4. Ans. (c)

Sol.

Find points of intersection by simultaneously solving for $x$ and $y$ from $y=mx+c$ and $x^{2}+y^{2}=a^{2}$ which comes out as $\left(–\frac{a^{2}m}{c},\frac{a^{2}}{c}\right).$

S5. Ans. (b)

Sol.

The length of tangent drawnj from any point on the circle $x^{2}+y^{2}+2gx+c=0$ to the circle $x^{2}+y^{2}+2gx+2fy+c\_{1}=0$ is $\sqrt{c\_{1}-c}$ given circles $\left(x-3\right)^{2}+\left(y+2\right)^{2}=5r^{2}$ and $\left(x-3\right)^{2}+\left(y+2\right)^{2}=r^{2}$

$∴$ length of tangent

= $\sqrt{9+4+r^{2}-9-4+5r^{2}}=\sqrt{4r^{2}}=2r=16⇒r=8$

$∴ $difference in area $π5r^{2}-πr^{2}=π4r^{2}=256π.$

S6. Ans. (b)

Sol.

Point is inside, outside or on the circle as $S\_{1}$ is $<, >,=0.$ For point $\left(-2, 1\right), S\_{1}<0.$

S7. Ans. (b)

Sol.

Here the equation of circle is

$\left(x-a\right)^{2}+\left(y-0\right)^{2}=a^{2}⇒x^{2}+y^{2}-2ax=0$

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Now the point of intersection of circle and chord $i.e.,$ Put $y=mx$ in equation of circle and solve it.

$O$ and $B$ are $O(0, 0)$ and $B\left(\frac{2a}{1+m^{2}},\frac{2am}{1+m^{2}}\right)$.

Hence the equation of circle (as chord $OB$ as diameter) is $\left(x^{2}+y^{2}\right)\left(1+m^{2}\right)-2a\left(x+my\right)=0.$

**Aliter :** Equation of circle $S+λL=0$

d$\left(x^{2}+y^{2}=2ax\right)+λ\left(y-mx\right)=0.$ Centre of this circle $\left(\frac{2a+λm}{2},\frac{λ}{2}\right)$ lies on $y=mx,$ we get $λ=-\frac{2am}{1+m^{2}}$ .

S8. Ans. (c)

Sol.

Let the mid-point of chord is $\left(h, k\right).$ Also radius of circle is 2. Therefore

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$\frac{OC}{OB}=\cos(45°⇒\frac{\sqrt{h^{2}+k^{2}}}{2}=\frac{1}{\sqrt{2}}⇒h^{2}+k^{2}=2)$

Hence, locus is $x^{2}+y^{2}=2.$

S9. Ans. (c)

Sol.

$T=S\_{1}$ is the equation of desired chord, hence

$xx\_{1}+yy\_{1}-a^{2}=x\_{1}^{2}+y\_{1}^{2}-a^{2}⇒xx\_{1}+yy\_{1}=x\_{1}^{2}+y\_{1}^{2}.$

S10. Ans. (b)

Sol.

Let the equations of circle are

 $S\_{1}≡x^{2}+y^{2}+2x+3y+1=0$

and $S\_{2}≡x^{2}+y^{2}+4x+3y+2=0$

Then the equation of common chord $PQ$ is

 $S\_{2}-S\_{1}=2x+1=0$

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Here, $C\_{1}\left(-1, -\frac{3}{2}\right), r\_{1}=\frac{3}{2}=C\_{1}P$ and $C\_{2}\left(-2, \frac{-3}{2}\right), r^{2}=\frac{\sqrt{17}}{2}$

$C\_{1}M=$ Perpendicular distance from $C\_{1}$ to the common chord $2x+1=0$

$⇒C\_{1}M=\frac{|-2+1|}{\sqrt{2^{2}}}=\frac{1}{2}$

Now,

$PQ=2PM=2\sqrt{\left(C\_{1}P\right)^{2}-\left(C\_{1}M\right)^{2}}=2\sqrt{\frac{9}{4}-\frac{1}{4}}=2\sqrt{2}$ .