Practise Questions - Part 1

1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a relation on the set A = {1, 2, 3, 4}. The relation R is
   (a) not symmetric (b) transitive
   (c) a function (d) reflexive [

2. The range of the function is $f(x) = {}^{7-x}P_{x-3}$ is
   (a) {1, 2, 3, 4} (b) {1, 2, 3, 4, 5, 6}
   (c) {1, 2, 3} (d) {1, 2, 3, 4, 5}

3. The domain of the function $f(x) = \dfrac{\sin^{-1}(x-3)}{\sqrt{9-x^2}}$ is
   (a) [1, 2] (b) [2, 3]
   (c) [2, 3] (d) [1, 2]

4. If $f: R \to S$ defined by $f(x) = \sin x - \sqrt{3}\cos x + 1$ is onto, then the interval of S is
   (a) [0, 1] (b) [–1, 1]
   (c) [0, 3] (d) [–1, 3]

5. The graph of the function $y = f(x)$ is symmetrical about $x = 2$ then
   (a) $f(x) = f(-x)$ (b) $f(2 + x) = f(2 - x)$
   (c) $f(x + 2) = f(x - 2)$ (d) $f(x) = -f(-x)$

6. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be a relation on the set A = {3, 6, 9, 12}
   The relation is
   (a) an equivalence relation
   (b) reflexive and symmetric only
   (c) reflexive and transitive only
   (d) reflexive only

7. A real valued function $f(x)$ satisfies the functional equation
   $$f(x - y) = f(x)f(y) - f(a - x)f(a + y)$$
   where a is a given constant and $f(0) = 1$, $f(2a - x)$ is equal to
   (a) $f(a) + f(a - x)$ (b) $f(-x)$
   (c) $-f(x)$ (d) $f(x)$

8. Let W denote the words in the English Dictionary. Define the relation R by $R = \{(x, y) \in W \times W$: the words x and y have at least one letter in common}, then R is
   (a) reflexive, not symmetric and transitive.
   (b) not reflexive, symmetric and transitive.
   (c) reflexive, symmetric and not transition.
   (d) reflexive, symmetric and transitive

9. The largest interval lying in $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ for which the function $f(x) = 4^{-x^2} + \cos^{-1}\left(\dfrac{x}{2} - 1\right) + \log \cos x$ is defined is
   (a) $\left(0, \dfrac{\pi}{2}\right)$ (b) $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$
   (c) $\left[-\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$ (d) $\left[0, \dfrac{\pi}{2}\right)$

10. Let **R** be the real line. Consider the following subsets of the plane **R** × **R**:
    $S = \{(x, y): y = x + 1 \text{ and } 0 < x < 2\}$
    $T = \{(x, y): x - y \text{ is an integer}\}$
    which of the following is true :
    (a) T is an equivalence relation on R but S is not
    (b) Neither S nor T is an equivalence relation.
    (c) Both S and T are equivalence relation on R.
    (d) S is an equivalence relation but T is not.

11. Let $f: N \to Y$ be a function defined as
    $f(x) = 4x + 3$ where $Y = \{y \in N : y = 4x + 3$ for some $x \in N\}$.
    Show that f is invertible and its inverse is
    (a) $g(y) = \dfrac{3y+4}{3}$ (b) $g(y) = 4 + \dfrac{y+3}{4}$
    (c) $g(y) = \dfrac{y+3}{4}$ (d) $g(y) = \dfrac{y-3}{4}$

12. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then
    (a) $B = C$ (b) $A \cap B = \phi$
    (c) $A = B$ (d) $A = C$

13. Consider the following relations
    $R = \{(x, y) \mid x, y$ are real numbers and $x = wy$ for some rational number $w\}$
    $S = \left\{\left(\dfrac{m}{n}, \dfrac{p}{q}\right)\right\}$ {m, n, p, q are integers such that $n \cdot q \neq 0$ and $qm = pn$}
    Then
    (a) S is an equivalence relation but R is not an equivalence relation
    (b) R and S both are equivalence relations.
    (c) R is an equivalence relation but S is not an equivalence relation.
    (d) Neither R nor S is an equivalence relation.

14. Let R be the set of real numbers.

Statement-1:

$A = \{ (x,y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement-2:

$B = \{ (x, y) \in R \times R : x = \alpha y$ for some rational numbers $\alpha\}$ in an equivalence relation on R.

15. Let f be a function defined by

$f(x) = (x - 1)^2 + 1, (x \geq 1)$

Statement-1:

The set $\{x : f(x) = f^{-1}(x)\} = \{1, 2\}$

Statement-2: f is a bijection and

$f^{-1}(x) = 1 + \sqrt{x-1}, x \geq 1.$

16. Consider the following relation R on the set of real square matrices of order 3.

$R = \{(A,B): A = P^{-1} B P$ for some invertible matrix $P\}$

Statement-1: R is an equivalence relation.

Statement-2: For any two invertible 3×3 matrices M and N, $(M N)^{-1} = N^{-1}M^{-1}$ **[2011]**

17. Let $X = \{1, 2, 3, 4, 5\}$. The number of different ordered pairs (Y, Z) that can be formed such that $Y \subseteq X$, $Z \subseteq X$ and $Y \cap Z$ is empty is

(a) 2^5 (b) 5^3
(c) 5^2 (d) 3^5.

18. Let A and B be two sets containing 2 elements and 4 elements respectively. The number of subsets of $A \times B$ having 3 or more elements is

(a) 211 (b) 256
(c) 220 (d) 219

19. Let P be the relation defined on the set of all real numbers such that

$P = \{(a, b) : \sec^2 a - \tan^2 b = 1\}$. Then P is

(a) reflexive and symmetric but not transitive.
(b) reflexive and transitive but not symmetric.
(c) symmetric and transitive but not reflexive.
(d) an equivalence relation.

20. Let $f(n) = \left[\frac{1}{3} + \frac{3n}{100}\right]n$, where $[n]$ denotes the greatest integer less than or equal to n. Then $\sum_{n=1}^{56} f(n)$ is equal to

(a) 689 (b) 1399
(c) 1287 (d) 56

21. The function $f(x) = |\sin 4x| + |\cos 2x|$ is a periodic function with period

(a) $\pi/2$ (b) 2π
(c) π (c) $\pi/4$

22. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by $f(x) = \frac{|x|-1}{|x|+1}$ then f is

(a) onto but not one-one
(b) both one-one and onto
(c) one-one but not onto
(d) neither one-one nor onto

23. A relation on the set $A = \{x: |x| < 3, x \in \mathbf{Z}\}$, where $\mathbf{Z}$ is the set of integers is defined by $R = \{(x, y) : y = |x|, x \neq -1\}$. Then the number of elements in the power set of R is

(a) 32 (b) 16
(c) 8 (d) 64

24. Let A and B be two sets containing four and two elements respectively. Then the number of subsets of the set $A \times B$, each having atleast three elements is

(a) 219 (b) 256
(c) 275 (d) 510

25. In a certain town, 25% of the families own a phone and 15% own a car, 65% families own neither a phone nor a car and 2000 families own both a car and a phone. Consider the following three statements:

(a) 5% families own both a car and a phone
(b) 35% families own either a car or a phone
(c) 40,000 families live in town. Then,

(a) only (a) and (b) are correct
(b) only (a) and (c) are correct
(c) only (b) and (c) are correct
(d) all (a), (b) and (c) are correct

26. Let $A = \{x_1, x_2, \ldots x_7\}$ and $B = \{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f : A \to B$ that are onto, if there exist exactly three x in A such $f(x) = y_2$ is equal to

(a) $14.{}^7C_2$ (b) $16.{}^7C_3$
(c) $12.{}^7C_2$ (d) $14.{}^7C_3$ **[2015, online]**

27. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and $S = \{x \in \mathbf{R} : f(x) = f(-x)\}$; then S

(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements

25. In a certain town, 25% of the families own a phone and 15% own a car, 65% families own neither a phone nor a car and 2000 families own both a car and a phone. Consider the following three statements:

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(d) all (a), (b) and (c) are correct

26. Let $A = \{x_1, x_2, \ldots x_7\}$ and $B = \{y_1, y_2, y_3\}$ be two sets containing seven and three distinct elements respectively. Then the total number of functions $f : A \to B$ that are onto, if there exist exactly three x in A such $f(x) = y_2$ is equal to

(a) $14.{}^7C_2$ (b) $16.{}^7C_3$
(c) $12.{}^7C_2$ (d) $14.{}^7C_3$ **[2015, online]**

27. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x, x \neq 0$ and $S = \{x \in \mathbf{R} : f(x) = f(-x)\}$; then S

(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements

28. If the function $f : [1, \infty[\to [1, \infty[$ is defined by $f(x) = 3^{x(x-1)}$, then $f^{-1}(x)$ is

(a) $\frac{1}{2}\left(1-\sqrt{1+4\log_3 x}\right)$ (b) $\frac{1}{2}\left(1+\sqrt{1+4\log_3 x}\right)$

(c) not defined 2(d) $\left(\frac{1}{3}\right)^{x(x-1)}$

29. For $x \in \mathbf{R}$, $x \neq 0$, $x \neq 1$, let $f_0(x) = \frac{1}{1-x}$ and $f_{n+1}(x) = f_0(f_n(x))$, $n = 0, 1, 2, \ldots$. Then the $f_{100}(3) + f_1\left(\frac{2}{3}\right) + f_2\left(\frac{3}{2}\right)$ is equal to

(a) $\frac{8}{3}$ (b) $\frac{4}{3}$

(c) $\frac{5}{3}$ (d) $\frac{1}{3}$

Practise Questions - Part 2

1. The domain of the function

$f(x) = \sqrt{2x-3} + \sin x + \sqrt{x-1}$ is

(a) $(-\infty, 1]$ (b) $[0, 1]$

(c) $\left[\frac{3}{2}, \infty\right)$ (d) $[1, \infty]$

2. Let $f : (1, \infty) \to (1, \infty)$ be defined by $f(x) = \frac{x+2}{x-1}$. Then

(a) f is 1 – 1 and onto
(b) f is 1 – 1 but not onto
(c) f is not 1 – 1 but onto
(d) f is neither 1 – 1nor onto

3. Let $A = \{(x, y) : x > 0, y > 0, x^2 + y^2 = 1\}$ and let $B = \{(x, y) : x > 0, y > 0, x^6 + y^6 = 1\}$ Then $A \cap B$

(a) A (b)B (c) ϕ
(d) $\{(0, 1), (1, 0)\}$

4. A school awarded 38 medals in football, 15 in basketball and 20 in cricket. Suppose these medals went to a total of 58 students and only three students got medals in all three sports. If only 5 students got medals in football and basketball, then the number of medals received in exactly two of three sports is

(a) 7 (b) 9
(c) 11 (d) 13

5. Let Q be the set of all rational numbers and R be the relation defined as

$R = \{(x, y) : 1 + xy > 0, x, y \in Q\}$

Then relation R is

(a) symmetric and transitive
(b) reflexive and transitive
(c) an equivalence relation
(d) reflexive and symmetric

6. The domain of the function $f(x) = \frac{1}{3 - \log_3(x-3)}$ is

(a) $(-\infty, 30)$ (b) $(-\infty, 30) \cup (30, \infty)$
(c) $(3, 30) \cup (30, \infty)$ (d) $(4, \infty)$

7. Let f : $\mathbf{R} \to \mathbf{R}$ be a function defined by $f(x) = x^{2009} + 2009x + 2009$

Then $f(x)$ is

(a) one-one but not onto
(b) not one-one but onto
(c) neither one-one nor onto
(d) one-one and onto

8. Let f be a function defined on $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ by

$f(x) = 3\cos^4 x - 6\cos^3 x - 6\cos^2 x - 3$

Then the range of $f(x)$ is

(a) $[-12, -3]$ (b) $[-6, -3]$
(c) $[-6, 3)$ (d) $(-12, 3]$

9. **Statement-1:** The function $f(x) = x^2 e^{-x^2} \sin|x|$ is even

Statement-2: Product of two odd function is an even function

10. Consider the following relations

R_1 = {(x, y) : x and y are integers and x = ay or y = ax for some integer a}

R_2 = {(x, y): x and y are integres and $ax + by = 1$ for some integers a, b}

Then

(a) R_1, R_2 are not equivalence relations
(b) R_1, R_2 are equivalence relation
(c) R_1 is an equivalence relation but R_2 is not
(d) R_2 is an equivalence relation but R_1 is not

11. Let f and g be functions defined by $f(x) = \frac{1}{x+1}$ $x \in \mathbf{R}$, $x \neq -1$ and $g(x) = x^2 + 1$, $x \in \mathbf{R}$. Then go f is

(a) one-one but not onto
(b) onto but not one-one
(c) both one-one and onto
(d) neither one-one nor onto

12. Let $\mathbf{N}$ be the set of natural number and for $a \in \mathbf{N}$, $a\mathbf{N}$ denotes the set $\{ax : x \in \mathbf{N}\}$.

If $b\mathbf{N} \cap c\mathbf{N}$, = $d\mathbf{N}$, where b, c, d are natural numbers greater than 1 and the greatest common divisor of b and c is 1 then d equals

(a) $\max\{b, c\}$ (b) $\min\{b, c\}$
(c) bc (d) $b + c$

13. Let $f(x) = (x + 1)^2 - 1$, $x \geq -1$, then the set $\{x : f(x) = f^{-1}(x)\}$:

(a) is an empty set
(b) contains exactly one element
(c) contains exactly two elements
(d) contains more than two elements

14. Let f : $\mathbf{R} \to \mathbf{R}$ be a function defined by

$f(x) = \frac{e^{|x|} - e^{-x}}{e^x + e^{-x}}$, then f is

(a) one-one and onto (b) one-one but not onto
(c) onto but not one-one (d) neither onto nor one-one

15. If f is a function of real variable x satisfying $f(x + 4) - f(x + 2) + f(x) = 0$, then f is periodic with period

(a) 8 (b) 10
(c) 12 (d) 6