# Exercise - Level 2

54. The range of the function $f(x) = \cos [x]$ for $-\pi/2 < x < \pi/2$ contains
(a) $\{-1, 1, 0\}$ (b) $\{\cos 1, 1, \cos 2\}$
(c) $\{\cos 1, -\cos 1, 1\}$ (d) $[-1, 1]$.

55. The domain of the function
$f(x) = \log_{10} \frac{x-5}{x^2-10x+24} - 3\sqrt{x+5}$ is
(a) $(-5, \infty)$ (b) $(5, \infty)$
(c) $(2, 5) \cup (5, \infty)$ (d) $(4, 5) \cup (6, \infty)$

56. If $g(x) = 1 + \sqrt[3]{x}$ then a function $f$ such that $f(g(x)) = 3 - 3(\sqrt[3]{x}) + x$ is
(a) $f(x) = x^3 - 3x^2 + x + 5$
(b) $f(x) = x^3 + 3x^2 - x - 5$
(c) $f(x) = x^3 - 3x^2 + 5$
(d) $f(x) = x^3 - 3x^2 + 3x + 3$

57. The function $f(x) = \frac{x}{e^x - 1} + \frac{x}{2} + 1$ is
(a) even (b) periodic
(c) odd (d) neither even nor odd

58. Let $f$ and $g$ be two functions defined by $f(x) = \frac{x}{x+1}$, $g(x) = \frac{x}{1-x}$. Then $(f \circ g)^{-1}(x)$ is equal to
(a) $x$ (b) 1
(c) $2x$ (d) none of these

59. Let $f : (-1, 1) \to (0, \pi)$ be defined by
$f(x) = \cot^{-1} \frac{2x}{1-x^2}$ Then
(a) $f$ is one-one but not onto
(b) $f$ is onto but not one-one
(c) $f$ is both one-one and onto
(d) $f$ is neither one-one nor onto

60. Let $f : X \to [1, 27]$ be a function by $f(x) = 5 \sin x + 12 \cos x + 14$. The set $X$ so that $f$ is one-one and onto is
(a) $[-\pi/2, \pi/2]$ (b) $[0, \pi]$
(c) $[0, \pi/2]$ (d) none of these

61. Let $f(x) = \frac{1-x}{1+x}$. Then $f \circ f(\cos x)$ is equal to
(a) $\cos 2x$ (b) $\cos x$
(c) $\tan 2x$ (d) $\tan x$

62. Let $f(x) = \sin\sqrt{x}$, then
(a) $f(x)$ is periodic with period $\sqrt{2\pi}$
(b) $f(x)$ is periodic with period $\sqrt{\pi}$
(c) $f(x)$ is periodic with period $4\pi^2$
(d) none of these

63. Let $X = Y = \mathbf{R} \sim \{1\}$. The function $f : X \to Y$ defined by $f(x) = \frac{x+2}{x-1}$ is
(a) one-one but not onto
(b) onto but not one-one
(c) neither one-one nor onto
(d) one-one and onto

64. If $f(x) = \frac{a^x + a^{-x}}{2}$ and $f(x + y) + f(x - y) = K f(x) f(y)$ then $K$ is equal to
(a) 2 (b) 4
(c) $-2$ (d) none of these

65. If $f(x) = 2 \sin^{-1} \sqrt{x-3}$, then the domain of $f$ is
(a) [3, 4] (b) $[-1, 1]$
(c) $[-\pi/2, \pi/2]$ (d) [6, 8]

66. Let $f(x) = x|x|$ and $g(x) = \sqrt{|x|}$ then the number of elements in the set $\{x \in \mathbf{R} : f(x) = g(x)\}$ is
(a) 1 (b) 2
(c) 3 (d) infinitely many

67. Which of the following functions are odd functions
(a) $f(x) = x\left(\frac{a^x + a^{-x}}{a^x - a^{-x}}\right)$
(b) $f(x) = \frac{a^x + x}{a^x - x}$
(c) $f(x) = \frac{a^x - 1}{a^x + 1}$
(d) $f(x) = x \log_2 \left(x + \sqrt{x^2 + 1}\right)$

68. Let $g(x) = 1 + x - [x]$ and $f(x) = \text{sgn } x$. Then for all $x$, $f \circ g(x)$ is equal to
(a) $x$ (b) 1
(c) $f(x)$ (d) $g(x)$

Note $f(x) = \text{sgn } x = 1$ if $x > 0$
$= 0$ if $x = 0$
$= -1$ if $x < 0$

69. The domain of definition of the function $f(x) = \log_2 (\log_{1/2} (x^2 + 4x + 3)) + \sin^{-1} (2[x]^2 - 3)$, $[x]$ denotes the greatest integer $\leq x$ is

(a) $0 < x \leq 1$ (b) $0 \leq x \leq 1$

(c) $-\infty < x \leq 0$ (d) none of these

70. If $g(f(x)) = |\sin x|$ and $f(g(x)) = (\sin \sqrt{x})^2$, then $f$ and $g$ may be given by

(a) $f(x) = \sin^2 x$, $g(x) = \sqrt{x}$

(b) $f(x) = \sin x$, $g(x) = |x|$

(c) $f(x) = x^2$, $g(x) = \sin x$

(d) $f$ and $g$ cannot be determined

71. If $f(x) = 3x - 5$, then $f^{-1}(x)$

(a) is given by $\dfrac{1}{3x-5}$

(b) is given by $\dfrac{x+5}{3}$

(c) does not exist because $f$ is not one-one

(d) does not exist because $f$ is not onto

72. Let $f(x) = |x - a|$, $a \neq 0$ then

(a) $f(x^2) = (f(x))^2$

(b) $f(|x|) = |f(x)|$

(c) $f(x + y) = f(x) + f(y)$

(d) none of these

73. If $n(A) = n(B) = 4$ then number of bijections from $A$ to $B$ is

(a) 6 (b) 24

(c) 12 (d) 18