Exercise - Concept Based Questions

1. Let U be a Universal set and $n(U) = 12$. If $A, B \subseteq U$ are such that $n(B) = 6$ and $n(A \cap B) = 2$ then $n(A \cup B')$ is equal to
   (a) 6 (b) 10
   (c) 7 (d) 8
2. Let R be a relation on $\mathbf{R}$ defined as $a\ R\ b$ if $|a| \le b$. Then, relation R is
   (a) reflexive (b) symmetric
   (c) transitive (d) not antisymetric
3. Let $f(x) = ax + b$, $x \in \mathbf{R}$, and $g(x) = x + d$, $x \in \mathbf{R}$, then $fog = gof$ if and only if
   (a) $f(a) = g(c)$ (b) $f(d) = g(b)$
   (c) $f(b) = g(d)$ (d) $f(c) = g(a)$
4. The domain of the function $f(x) = \dfrac{\sin x}{\sqrt{|x| - x}}$ is
   (a) $\mathbf{R}$ (b) $\mathbf{R} \sim \{0\}$
   (c) $\mathbf{R}^+$ (d) $\mathbf{R}^-$
   ($\mathbf{R}^+$ is the set of positive real numbers and $\mathbf{R}^-$ is the set of negative real numbers)
5. The domain of $y = \cos^{-1}(1 - 2x)$ is
   (a) $[-1, 1]$ (b) $[0, 1]$
   (c) $[-1, 0]$ (d) $\left[-\dfrac{1}{2}, \dfrac{1}{2}\right]$
6. If $f(x) = \sin x - \cos x$ is written as $f_1(x) + f_2(x)$ where $f_1(x)$ is even and $f_2(x)$ is odd then
   (a) $f_1(x) = \cos x$ (b) $f_1(x) = -\cos x$
   (c) $f_2(x) = -\sin x + \cos x$ (d) $f_2(x) = \sin(2\pi - x)$
7. The range of $y = 1 - \sin x$ is
   (a) $[-1, 1]$ (b) $[0, 1]$
   (c) $[-1, 2]$ (d) $[0, 2]$
8. The function $f(x) = x^2 \log \dfrac{1 - x}{1 + x}$ is
   (a) a periodic function (b) a bounded function
   (c) an odd function (d) an even function

Exercise - Level 1

9. If $A = \{2, 3, 4, 5, 7\}$, $B = \{1, 2, 4, 7, 9\}$ then $((A \sim B) \cup (B \sim A)) \cap A$ is equal to
   (a) $\{3, 5\}$ (b) $\{2, 4\}$
   (c) $\{3, 7\}$ (d) $\{2, 7\}$
10. If $A = \{2x : x \in N\}$, $B = \{3x : x \in N\}$ and $C = \{5x : x \in N\}$ then $A \cap (B \cap C)$ is equal to
    (a) $\{15, 30, 45, ...\}$
    (b) $\{10, 20, 30 ...\}$
    (c) $\{30, 60, 90, ...\}$
    (d) $\{7, 14, 21 ...\}$.
11. If X and Y are two sets such that $X \cap Y = X \cup Y$, then
    (a) $X \subset Y, X \ne Y$ (b) $Y \subset X, Y \ne X$
    (c) $X = Y$ (d) none of these
12. If X, Y and A are three sets such that $A \cap X = A \cap Y$ and $A \cup X = A \cup Y$ then
    (a) $X \subset Y$ (b) $Y \subset X$
    (c) $X = Y$ (d) none of these
13. If $A = \{x : x \in R \text{ and satisfy } x^2 - 15x + 56 = 0\}$
    $B = \{x : x \in N \text{ and } x + 5 \le 14\}$
    and $C = \{x : x \in N \text{ and } x/112\}$.
    Then $A \cup (B \cap C)$ is equal to
    (a) $\{1, 2, 3, ...8\}$ (b) $\{8, 7\}$
    (c) $\{1, 2, 8, 7\}$ (d) $\{1, 2, 4, 7, 8\}$

14. If A is the set of letters needed to spell "MATHEMATICS" and B is the set of letters needed to spell STATISTICS, then

(a) $A \subset B$ (b) $A \sim B = \phi$
(c) $A \Delta B = A \sim B$ (d) none of these

15. If A and B are two sets such that $n(A \cup B) = 36$, $n(A \cap B) = 16$ and $n(A \sim B) = 15$, then $n(B)$ is equal to

(a) 21 (b) 31
(c) 20 (d) 52

16. The maximum number of sets obtainable from A and B by applying union and difference operations is

(a) 5 (b) 6
(c) 7 (d) 8

17. If A and B both contain same number of elements and are finite sets then

(a) $n(A \cup B) = n(A \cap B)$ (b) $n(A \sim B) = n(B \sim A)$
(c) $n(A \Delta B) = n(B)$ (d) $n(A \sim B) = n(A)$

18. If $A \Delta B = A \cup B$ then

(a) $A = B$ (b) $A \cap B = \phi$
(c) $A \Delta B = \phi$ (d) $A \Delta B = A \sim B$

19. In a class 60% passed their Physics examination and 58% passed in Mathematics. Atleast what percentage of students passed both their Physics and Mathematics examination?

(a) 18% (b) 17%
(c) 16% (d) 2%

20. If the relation $R: A \to B$, where $A = \{1, 2, 3\}$ and $B = \{1, 3, 5\}$ is defined by
$R = \{(x, y): x < y, x \in A, y \in B\}$, then

(a) $R = \{(1, 3), (1, 5), (2, 3), (2, 5), (3, 5)\}$
(b) $R = \{(1, 1), (1, 5), (2, 3), (3, 5)\}$
(c) $R^{-1} = \{(3, 1), (5, 1), (3, 2), (5, 3)\}$
(d) $R^{-1} = \{(1, 1), (5, 1), (3, 2), (5, 3)\}$

21. The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by

$$R = \{(x, y): |x^2 - y^2| < 16\}$$

is given by

(a) $R_1 = \{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$
(b) $R_2 = \{(2, 2), (3, 2), (4, 2), (2, 4)\}$
(c) $R_3 = \{(3, 3), (4, 3), (5, 4), (3, 4)\}$
(d) none of these

22. Let L be the set of all lines in a plane and R be a relation on L defined by $l_1 R\ l_2$ if and only if $l_1 \perp l_2$ then R is

(a) reflexive (b) symmetric
(c) transitive (d) an equivalence relation

23. For non-empty subsets A and B,

(a) Any subset of $A \times B$ defines a function from A to B.
(b) Any subset of $A \times B$ defines an equivalence relation
(c) Any subset of $A \times A$ defines a function on A
(d) none of these.

24. Let $f(x) = (x + 1)^2 - 1$ $(x \geq -1)$. Then the set $S = \{x : f(x) = f^{-1}(x)\}$ contains

(a) $\left\{0, -1, \dfrac{-3 + i\sqrt{3}}{2}, \dfrac{-3 - i\sqrt{3}}{2}\right\}$
(b) $\{0, 1, -1\}$
(c) $\{0, -1\}$
(d) none of these

25. If a set A has n elements then the number of all relations on A is

(a) 2^{n^2} (b) $2^{n^2} - 1$
(c) 2^n (d) none of these

26. The function f: $\mathbf{R} \to \mathbf{R}$ given by $f(x) = 3 - 2 \sin x$ is

(a) one-one (b) onto
(c) bijective (d) none of these

27. Which of the following are functions?

(a) $\{(x, y): y^2 = 4ax, x, y \in \mathbf{R}\}$
(b) $\{(x, y): y = |x|, x, y \in \mathbf{R}\}$
(c) $\{(x, y): x^2 + y^2 = 1, x, y \in \mathbf{R}\}$
(d) $\{(x, y): x^2 - y^2 = 1, x, y \in \mathbf{R}\}$

28. If f: $\mathbf{R} \to \mathbf{R}$ defined by $f(x) = x^4 + 2$ then the value of $f^{-1}(83)$ and $f^{-1}(-2)$ respectively are

(a) ϕ, $\{3, -3\}$ (b) $\{3, -3\}$, ϕ
(c) $\{4, -4\}$, ϕ (d) $\{4, -4\}$, $\{2, -2\}$.

29. The minimum number of elements that must be added to the relation $R = \{(1, 2), (2, 3)\}$ on the sub set $\{1,2,3\}$ of natural numbers so that it is an equivalence relation is

(a) 4 (b) 7
(c) 6 (d) 5

30. Let X be a non-empty set and $P(X)$ be the set of all subsets of X. For $A, B \in P(X)$, ARB if and only if $A \cap B = \phi$ then the relation

(a) R is reflexive
(b) R is symmetric
(c) R is transitive
(d) R is an equivalence relation

31. Let f be a function satisfying $2f(x) - 3f(1/x) = x^2$ for any $x \neq 0$. Then the value of $f(2)$ is

(a) -2 (b) $-7/4$
(c) $-7/8$ (d) 4

32. If $f(x) = \begin{vmatrix} 1 & x & (x+1) \\ 2x & x(x-1) & (x+1)x \\ 3x(x-1) & x(x-1)(x-2) & x(x-1)(x+1) \end{vmatrix}$

then

$f(50) + f(51) + \ldots\ldots + f(99)$ is equal to

(a) 0 (b) 1275
(c) 3725 (d) none of these

33. The domain of definition of the functions $y(x)$ given by the equation $a^x + a^y = a$ $(a > 1)$ is

(a) $0 < x \le 1$ (b) $0 \le x \le 1$
(c) $-\infty < x < 1$ (d) $-\infty < x \le 0$

34. Let $f(x) = \dfrac{x-[x]}{1+x-[x]}$, where $[x]$ denotes the greatest integer less than or equal to x, then the range of f is

(a) $[0, 1/2]$ (b) $[0, 1)$
(c) $[0, 1/2)$ (d) $[0, 1]$

35. If $2f(x^2) + 3f(1/x^2) = x^2 - 1$, then $f(x^2)$ is

(a) $(1 - x^4)/5x^2$ (b) $(1 - x^2)/5x$
(c) $5x^2/(1 - x^4)$ (d) none of these

36. If $f(x + 3y, x - 3y) = 12xy$, then $f(x, y)$ is

(a) $2xy$ (b) $2(x^2 - y^2)$
(c) $x^2 - y^2$ (d) none of these

37. If the function $f : [1, \infty) \to [1, \infty)$ is defined by $f(x) = 2^{x(x-1)}$ then $f^{-1}(x)$ is

(a) $(1/2)^{x(x-1)}$ (b) $\frac{1}{2}\left(1+\sqrt{1+4\log_2 x}\right)$
(c) $\frac{1}{2}\left(1-\sqrt{1+4\log_2 x}\right)$ (d) not defined

38. If $n(A) = 3$ and $n(B) = 5$ then number of one-one functions that can be defined from A to B is

(a) 30 (b) 40
(c) 120 (d) 60

39. Let f: $\{x, y, z\} \to \{1, 2, 3\}$ be a one-one function. If it is given that exactly one of the following statements is true,

Statement-1: $f(x) = 1$, Statement-2: $f(y) \ne 1$, Statement-3: $f(z) \ne 2$.

then $f^{-1}(1)$ is

(a) x (b) y
(c) z (d) none of these

40. The value of $n \in \mathbf{z}$ for which the function

$f(x) = \dfrac{\sin nx}{\sin(x/n)}$ has 4π as its period is

(a) 2 (b) 3
(c) 4 (d) 5

41. Let R be a relation on $\mathbf{N}$ defined by

$$R = \{(m, n): m, n \in N \text{ and } m = n^2\}.$$

Which of the following is true.

(a) $(n, n) \in R$. $\forall\, n \in \mathrm{N}$
(b) $(m, n) \in R \Rightarrow (n, m) \in R$
(c) $(m, n) \in R, (n, p) \in R \Rightarrow (m, p) \in R$
(d) none of these

42. Of the number of three athletic teams in a school, 21 are in the basketball team, 26 in hockey team and 29 in the football team, 14 play hockey and basketball, 15 play hockey and football, 12 play football and basketball and 8 play all the games. The total number of members is

(a) 42 (b) 43
(c) 45 (d) none of these

43. E, I, R, O denote respectively the sets of all equilateral, isosceles, right angled and obtuse angled triangles in a plane, then which of the following is not true.

(a) $R \cap E = \phi, R \cap I \ne \phi$
(b) $E \cap O = \phi, O \cap I \ne \phi$
(c) $E \cap I = \phi, E \cap O \ne \phi$
(d) $E \cap I \ne \phi, E \subset I$

44. If $A = \{3^n : n \in \mathbf{N}, n \le 6\}$, $B = \{9^n : n \in \mathbf{N}, n \le 4\}$ then which of the following is false

(a) $A \,\Delta\, B = \{6561\}$
(b) $A \sim B = \{3, 27, 243\}$
(c) $A \cap B = \{9, 81, 729\}$
(d) $A \cup B = \{3, 9, 27, 81, 243, 729, 6561\}$

45. If $f : \mathbf{R} \to \mathbf{R}$ given by $f(x) = ax + \sin x + a$, then f is one-one and onto for all

(a) $a \in \mathbf{R}$ (b) $a \in \mathbf{R} \sim [-1, 1]$
(c) $a \in \mathbf{R} \sim \{0\}$ (d) $a \in \mathbf{R} \sim \{-1\}$

46. If f: $(0, \pi) \to \mathbf{R}$ is given by $f(x) = \sum_{k=1}^{n}[1+\sin kx]$, $[x]$ denotes the greatest integer function, then the range of $f(x)$ is

(a) $\{n-1, n+1\}$ (b) $\{n\}$
(c) $\{n, n+1\}$ (d) $\{n-1, n\}$

47. If the number of elements in $(A \sim B) \sim C$, $(B \sim C) \sim A$, $(C \sim A) \sim B$ and $A \cap B \cap C$ is 10, 15, 20, and 5 respectively then the number of elements in $(A \Delta B) \Delta C$ is

(a) 35 (b) 50
(c) 40 (d) 45

48. Let $f(x) = \dfrac{x-3}{x+1}, x \ne -1$. Then $f^{2010}(2014)$ (where $f^n(x) = fof\ .\ .\ .$ of (x) (n times)) is

(a) 2010 (b) 4020
(c) 4028 (d) 2014

Exercise - Reasoning Type Questions

49. Let $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ be a relation on set $A = \{1, 2, 3\}$

    Statement-1: R is not an equivalence relation on A.
    Statement-2: R is function from A to A.

50. **Statement-1:**

    If A is a set with 5 elements and B is a set containing 9 elements, then the number of injective mappings from A to B to $9! - 4!$.

    Statement-2: The total number of injective mappings from a set with m elements to a set with n elements, $m \le n$, is $\dfrac{n!}{(n-m)!}$.

51. Let $f(x) = \sin x + \cos x$, $g(x) = \dfrac{\sin x}{1 - \cos x}$.

    Statement-1: f is neither an odd function nor an even function.

    Statement-2: g is an odd function.

52. **Statement-1:** A function $f : R \to R$ satisfies the equation $f(x) - f(y) = x - y \ \forall \ x, y \in R$ and $f(3) = 2$, then $f(xy) = xy - 1$

    Statement-2: $f(x) = f(1/x) \ \forall \ x \in R, x \ne 0$, and

    $f(2) = 7/3$ if $f(x) = \dfrac{x^2 + x + 1}{x^2 - x + 1}$.

53. **Statement-1:** Let $A = \{2, 3, 7, 9\}$ and $B = \{4, 9, 49, 81\}$ $f : A \to B$ is a function defined as $f(x) = x^2$. Then f is a bijection from A to B.

    Statement-2: A function f from a set A to a set B is a bijection if $f(A) = B$ and $f(x_1) \ne f(x_2)$ if $x_1 \ne x_2$ for all $x_1, x_2 \in A$ and $n(A) = n(B)$.