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GROUP

RAILWAY RECRUITM

QUANTITATIVE

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Contents

1. Number System	01
2. Fraction & Decimal	20
3. Surds & Indices	33
4. HCM & LCM	54
5. Algebra	71
6. Trigonometry	109
7. Height & Distance	137
8. Geometry	140
9. Mensuration	178
10. Static	190

NUMBER SYSTEM

Introduction

Numbers

Real Numbers
- Which Can be represented on number line

eg: $-1, 0, 1, 2\cdot3, 5/7$

Rational Number
- Which Can be written in form of p/q $(q \neq 0)$

eg: $-2/3, 5/7, 4, 1, \sqrt{16}$

Fraction/ Decimal
- Which are in form of p/q or having Decimal

Integer
- A Complete Number Which is not a fraction or decimal.

eg: $-3, -71, -0, 1, 5, 243$

[It Can be Positive, negative or zero]

Irrational Number
- Which Can not be written in form of p/q.

eg: $\sqrt{5}, \sqrt{23}, \pi, e$

$\pi \Rightarrow$ Pie $= 3\cdot1457$------

$e \Rightarrow 2\cdot73828$--------

Imergenary Numbers
- Which Can be not refresented On Number line

eg: i, $a + ib$

$i = i0 + a = \sqrt{-1}$

Such number no existin real

- **Whole Numbers:** Integers Starting from 0.
- **Natural Numbers:** Integers Starting from 1.
- **Prime Numbers:** The number which is divisible by 1 & no. itself is Called a Prime number.

eg: 2, 3, 5, 7, 11, 13 etc.

1 is not a Prime number

There are 25 Prime number b/w 1 to 100

- **Composite Number:** The number which have more than two factors are Called Composite numbers.

eg: 4, 6, 12, 21, 28 etc.

The numbers which are not prime are Composite Number

Co-Prime Number: Numbers having their HCF is 1 are termed as Co-prime Numbers.

eg: 14 & 15.

Even Number: Rational number which are the multiple of 2 is Called as even numbers.

eg: 2, 4, 6, 48, 92 ..... etc.

Odd Number: Rational Numbers which are not multiple of 2 are Odd Number.

eg: 1, 3, 5, 91, 103, 249 .....

even Numbers ending digit is 2, 4, 6, 8, 0 & Odd Numbers ending digit is 1, 3, 5, 7, 9

Properties of Odd and even Numbers:

- even + even = Even
- ODD + ODD = Even
- Even + ODD = ODD
- Even + Even ---------- + n times = Even (always)
- Odd + Odd ---------- odd numbers of times = ODD
- ODD + ODD ---------- even number of times = Even
- Even x Even = Even
- Eved x odd = Even
- Odd x odd = Odd
- Even x (Even / odd) = Even

Decimal

Terminating
- Decimal Vanishes after Some places

These No's Can be represented by– $1.\overline{23}$

Non-terminating
Eg: 1.23456789......
1.111111 ......
(Decimal never Vanishes)

Recurring
Digit of Decimal is repeat after Some decimal Places
Eg: 1.23232323.......

These are rational Numbers

Non Recurring
- Digit of Decimal does not repeat

Eg: 1.233759257.

These are Irrational Number

Converting Recurring in P/q Form:
(Solving the – (Bar) Problems)

Eg: $x = 0.\overline{7}$, Convert x into P/q form.

Solⁿ ⇒ $x = 0.7777........$ –①

$10x = 7.7777........$ –②

$9x = 7.0000$

$\boxed{x = 7/9}$

- if (–) on one digit = Multiply by 10
- if (–) on two digit → multiply by 100

Tricks

Type-1

(a) $x = 0.\overline{8}$

$x = \frac{8}{9}$ → As many digits Contain ('−'), Write 9 as many times;-

(b) $x = 0.\overline{78}$

$x = \frac{\cancel{78}}{\cancel{99}} = \frac{26}{33}$ Ans

Type-II

(c) $x = 0.3\overline{84}$

$= \frac{384-3}{990}$ → Number After Decimal − Number not Contain bar
→ 9 as many digit in (−), & 0 as many times not Contain (−),

$= \frac{381}{990} = \frac{127}{330}$ Ans

$x = 52\overline{48}$

$= \frac{5248-52}{9900} = \frac{5196}{9900}$ $\frac{1732}{3300}$ Ans

Type-III

(a) $2.6\overline{5}$

$\Rightarrow 2+0.6\overline{5}$

$= 2+\frac{65-6}{90}$ (Same as type II)

$= 2+\frac{59}{90} = \frac{239}{90}$ Ans

(b) $5.\overline{95}$

$= 5+0.\overline{95}$

$= 5+\frac{95}{99}$

$= \frac{590}{99}$

5

Divisibility Rules :-

NUMBER	RULE	EXAMPLE
2	Last digit is divisible by 2, or last digit is 0, 2 4, 6, 8.	Eg: 2348 1948
3	Sum of digit is divisible by 3.	Eg: 1071 1+0+7+1=9
4	last two digit of number is divisible by 4	14<u>32</u> 92<u>84</u>
5	Last digit is 5 or 0	2335, 1990
6	Number is divisible by 2 and 3 each	132 → divisible by 2 1+3+2 → divisible 3
7	• Multiply last digit by 5 • Add the above number • If remaing digits divisible by 7, then number is divided by 7	Eg: 343 (i) 3 x 5 = 15 34 − 15 = [redacted] 49 divisible by 7.
8	Last 3 digit are divisible by 8	8032 → 32 Divisible by 8.
9.	Sum of digits is divisible by 9	1071 → 1+0+7+1 = 9 divisible by 9
11.	• Defrence of Sum of digit at odd Places & Sum of digit at even places.	• 1331 (3+1) − (3+1) = 0 • 11718520 (1+7+8+2) − (1+1+5+0) = 11

(2) If $3x2680$, is divisible by 11, then the Value of x is :

Soln: (Sum of Odd Place digit) − (Sum of Even Place digit)

$= (3+2+8) - (x+6+0)$

$= 13 - 6 - x$

$= 7 - x$ (Either 0 or divisible by 11)

$= 7 - x = 0$

$\boxed{x = 7}$ Ans.

Cyclicity:

Unit digit is repeated after Some time of an exponent.

$2^1=2$ $2^2=4$ $2^3=8$ $2^4=16$ $2^5=32$ $2^6=64$	$3^1=3$ $3^2=9$ $3^3=27$ $3^4=81$ $3^5=243$ $3^6=729$	$4^1=4$ $4^2=16$ $4^3=64$ $4^4=216$ Cyclicity=2	$7^1=7$ $7^2=49$ $7^3=343$ $7^4=2401$ $7^5=16807$ Cyclicity =4
Cyclicity =4	Cyclicity = 4		
$8^1=8$ $8^2=64$ $8^3=512$ $8^4=4096$ $8^5=32768$	$9^1=9$ $9^2=81$ $9^3=729$ $9^4=6561$ Cyclicity=2		
Cyclicity = 4			

Eg: $(2)^{423}$, Find the digit at units Place

Soln (a) divide the power by 4

In Exams divide in Mind, not in Pen-Paper.

$$
\begin{array}{r}
4\overline{)\,423\,}(105 \\
4 \\
\hline
23 \\
20 \\
\hline
3
\end{array}
$$

Remainder = 3

$2^3 = 8$ Ans

Unit digit and ten's digit Concept-

★ 1 2 3 4

4 → Unit digit

3 → Ten's digit

Eg: $\boxed{\text{Type-I}}$

(a) $29 \times 45 = 9 \times 5 = 45$ Unit digit $= 5$

(b) $18 \times 18 \times 18 + 3$

$8 \times 8 \times 8 + 3$

64×8

$32 + 3 = 35 \Rightarrow = 5$

$\boxed{\text{Type = II}}$

(a) (0, 1, 2, 3, 4, 5, 6, 7, 8, 9)

↓

Cyclicity Concept

(b) (0, 1, 5, 6)

↓

If there number are at unit place unit digit of multiplication is also a Same number.

Eg; (a) 35×35 (b) 36×96

1225 → Same $= 3456$ → Same

Helping Hand ÷

(a) Divide the Power by 4.

(b) Remainder of division is 0, 1, 2, 3...

(c) Remainder ⇒ 1 = x^1 is unit digit

Remainder ⇒ 2 = x^2 is unit digit

Remainder ⇒ 3 = x^3 is unit digit

Remainder ⇒ 0 = x^4 is unit digit

If x^4 is 2 or 3 digit number, then unit digit of that number, will be the unit digit of original Exponent.

Solved Examples

① What least number must be added to 1056, So that Sum is Completely divisible by 23?

Soln ⇒

```
23) 1056 (45
    92
    ---
     136
     115
     ---
      21
```

then Number added is $= 23 - 21$

$= 2$ Ans

② The largest 4 digit number exactly divisible by 88 is –

(a) 9944 (b) 9768 (c) 9988 (d) 8888

Soln ⇒ Largest 4 digit Number $= 9999$

```
88) 9999 (113
    88
    ---
    119
     88
    ---
     319
     264
     ---
      55
```

55 ⟶ Subtract from the 4 digit largest number.

$= 9999 - 55 = 9944$ Ans

③ If the number 517 H 324 is Completely divisible by 3, then the Smallest whole no. in place of H will be.

(a) 0 (b) 1 (c) 2 (d) None

$5+1+7+H+3+2+4$
$= 22 + H$

> If number is divisible by 3, then sum of digit is also divisible by 3.

If 2 is used in place of H, then number is divisible by 3 (i.e. 24)

④ Which one of the fallowing no. is divisible by 11?

(a) 235641 (b) 245642 (c) 315624 (d) 415624

Soln ⇒ (a) 235641
$(2+5+4) - (3+6+1) = 1$ (not divisible by 11)

(b) 245642
$(2+5+4) - (4+6+2) = 1$ (not divisible by 11)

(c) 315624
$(3+5+2) - (1+6+4) = -1$ (not divisible by 11)

(d) 415624
$(4+5+2) - (1+6+4) = 0$ (divisible by 11)

> If a number is divisible by 11, the Difference of Sum of digit at odd places & Sum of digit at even places is either 0 or divisible by 11.

⑤ Which on the fallowing number is divisible by 24 -

Soln ⇒ (a) 35718 (b) 63810 (c) 63810 (c) 537804 (d) 3125736

	③	⑧
35718	$3+5+7+1+8 = 24$ ✓	718 ✗
63810	$6+3+8+1+0 = 18$ ✓	810 ✗
537804	$5+3+7+8+0+4 = 27$ ✓	804 ✗
3125736	$3+1+2+5+7+3+6 = 27$ ✓	736 ✓ ✓

> If a no. is divisible by another number then it must be divisible by it's prime factors.

Unit digit Concept:

⑥ The digit at unit's place of the Product-

$81 \times 82 \times 83 \ldots\ldots \times 89$ is

(a) 0 (b) 2 (c) 6 (d) 8

Soln ⇒ $81 \times 82 \times 83 \times 84 \times 85 \ldots \times 89$

$1 \times 2 \times 3 \times 20 \ldots \times 6 \times 7 \times 8 \times 9$
$= 0$

If we multiply a number by 0, the result at unit place is always zero.

⑦ The digit in unit's Place of the Product $(2153)^{167}$ is:

(a) 1 (b) 3 (c) 7 (d) 9

Soln ⇒ 215<u>3</u> → Let base is 3

ⓑ $\frac{167}{4}$ ⇒ Remainder is 3

ⓒ $3^3 = 27$ → unit digit is 7

⑧ Unit digit in $(264)^{102} + (264)^{103}$ is -

ⓐ 0 (b) 4 (c) 6 (d) 8

Soln ⇒ $(264)^{102} + (264)^{103}$

$= 6 + 4$
$= 10$
unit digit = 0

> If Base is 4, then
> (a) ⇒ Unit digit of even power is always 6
> (b) ⇒ Unit digit of odd Power is always 4.
> because Cyclicity is 2

(9) Unit digit of $(169)^{537} + (94)^{394}$ is.

(a) (b) (c) (d)

Soln ⇒ $(169)^{537} + (94)^{394}$

$= 9 + 6$

$= 15$

= unit digit is 5 Ans

If the Base is 9
(a) Unit digit of ODD Power is always 9.
(b) unit digit of even Power is always 1.
because Cyclicity is 2.

(10) The digit in the unit place of
$[(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + 259 + (73)$ is —

(a) 1 (b) 4 (c) 5 (d) 6

Soln ⇒ $(251)^{98} + (21)^{29} - (106)^{100} + (705)^{35} - (16)^4 + 259 + (73)^{51}$

$1 + 1 - 6 + 5 - 6 + 9 + 7$

Unit digit of base 1, 5, 6, is always Same

$\frac{51}{3}$ = Remainder 3
$3^3 = 27$

$= 23 - 12 = 11$ Ans

(11) Unit digital in expression of $(2137)^{754}$ is —

(a) 1 (b) 3 (c) 7 (d) 9

Soln ⇒ $(2137)^{754}$ → Base is 7

$\frac{754}{4}$ Remainder = 2

$7^2 = 49$ → unit digit is 9 ✓

(12) Find the unit's digit of $(358)^{64} - (253)^{36}$.

(a) 5 (b) 4 (c) 7 (d) 9

Soln ⇒ $(358)^{64} - (253)^{36}$

$\frac{64}{8}$ $\frac{36}{3}$

0 ⟶ Remainder ← 0 ⟶ 3^4

$8^4 = 64 \times 64 = 16 - 1$

⇒ 6 − 1
= 5 Ans

Solved Examples

1- What least number must be added to 1056, so that sum is completely divisible by 23?

(a) 2 (b) 2 (c) 18 (d) 21

Sol.

$$23 \overline{)1056}(45$$

92

136

115

21

then number added is

$= 23-21$

$= 2.$

2- The largest 4 digit number exactly divisible by 88 is-

(a) 9944 (b) 9768 (c) 9988 (d) 8888

Sol.

Largest 4 digit Number = 9999

$$88 \overline{)9999}(113$$

88

119

88

319

264

55 ⟶ Subtract from the 4 digit Largest number

$= 9999 - 55$

$= 9944.$

3- If the number 517x324 is completely divisible by 3, them the smallest whole no. in place of x will be-

(a) 0 (b) 1 (c) 2 (d) None

Sol.

$5+1+7+H+3+2+4$

$= 22+H$

If number in divisible by 3 them sum of digit is also divisible by 3.

If 2 is used in place of H, then number is divisble by 3 (i.e. 24)